Minimal Lepton Flavor Violation in Randall-Sundrum Model

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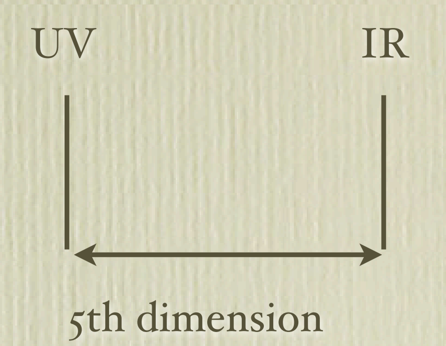
Introduction

Randall & Sundrum, 1999

- Randall-Sundrum model: solution to gauge hierarchy problem

5D metric: $ds^2 = e^{-2kr_c\phi}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 d\phi^2$

$v_{ew} \sim e^{-\pi k R} M_{pl}$



- Precision Constraints
  - expand the bulk gauge symmetry to $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
    - custodial symmetry preserved
    - 1st KK mass ~ 3 TeV allowed by EWPT
- Constraints on flavor sector:
  - model independent analysis: with O(1) couplings:

$\Lambda > (10^2\text{-}10^3)$ TeV

Parameter	Limit on Λ_F (TeV)
$\mathrm{Re}C_K^1$	$1.0 \cdot 10^3$
$\mathrm{Re}C_K^4$	$12 \cdot 10^3$
$\mathrm{Re}C_K^5$	$10 \cdot 10^3$
$\mathrm{Im}C_K^1$	$15 \cdot 10^3$
$\mathrm{Im}C_K^4$	$160 \cdot 10^3$
$\mathrm{Im}C_K^5$	$140 \cdot 10^3$

Flavor Sector

- SM fermions on the TeV brane
  - cutoff scale ~ 1 TeV
  - leads to dangerously large FCNC

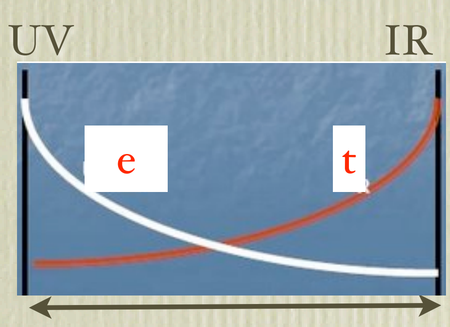
- SM fermions and gauge fields in the bulk

Gherghetta & Pomarol, 2000

  - generate fermion mass hierarchy by wave function localization

$$\psi_{(0)} \sim e^{(1/2-c)ky}$$

5D Bulk mass term



  - two sources of flavor violations:
    - 5D Yukawa coupling constants
    - 5D bulk mass terms
    - generally independent

Flavor Violations

- 5D Lagrangian

$$\mathcal{L}_{5\mathrm{D}} \supset \overline{\Psi} C_\Psi \Psi + \overline{\psi_u} C_{\psi_u} \psi_u + \overline{\psi_d} C_{\psi_d} \psi_d + H\,\overline{\Psi} \lambda_U \psi_u + \overline{H}\,\overline{\Psi} \lambda_D \psi_d$$

- unitary transformations to diagonal basis for bulk mass matrices
- decomposition of fermion field $\psi(x,y) = \sum_n \frac{e^{2kr_c|\phi|}}{\sqrt{r_c}} \psi_n(x) f_n(\phi, c)$
- couplings between zero mode fermions and gauge boson KK modes:

$$\sum_n G^n ({\Psi^0}^\dagger f^2_{\Psi^0} \Psi^0 + {\psi_u^0}^\dagger f^2_{\psi_u^0} \psi_u^0 + {\psi_d^0}^\dagger f^2_{\psi_d^0} \psi_d^0)$$

Flavor Violations

- effective 4D Yukawa interactions

$$H\Psi^0 f^\dagger_{\Psi^0}\lambda_u^{5D} f_{\psi_u^0}\psi_u^0 + \overline{H}\Psi^0 f^\dagger_{\Psi^0}\lambda_d^{5D} f_{\psi_d^0}\psi_d^0$$

- effective 4D Yukawa couplings

$$\lambda_U^{4D} = f^\dagger_{\Psi^0}\lambda_U f_{\psi_u^0},\ \lambda_D^{4D} = f^\dagger_{\Psi^0}\lambda_D f_{\psi_d^0}$$

  - diagonalized by the following chiral rotations:

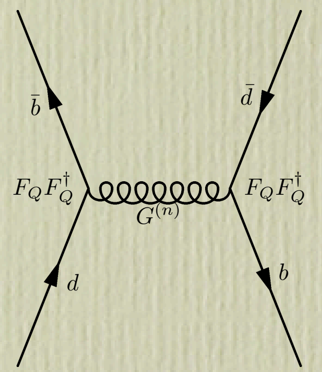
$$\Psi^0 \to V\Psi^0,\ \ \psi_u^0 \to W_u\psi_u^0,\ \ \psi_d^0 \to W_d\psi_d^0$$

- in mass eigenstates of SM fermions: fermion-gauge couplings

$$\sum_n G^n({\Psi^0}^\dagger V^\dagger f^2_{\Psi^0} V\Psi^0 + {\psi_u^0}^\dagger W_u^\dagger f^2_{\psi_u^0} W_u\psi_u^0 + {\psi_d^0}^\dagger W_d^\dagger f^2_{\psi_d^0} W_d\psi_d^0)$$

  - non-universal f : leads to tree-level FCNCs
  - RS-GIM mechanism:
    - light fermions close to Planck brane
    - gauge zero-mode flat
    - somewhat alleviate flavor constraints, though not enough

Flavor Violations in Quark Sector



Contributions to $\Delta F = 2$ processes from KK gluon exchange.

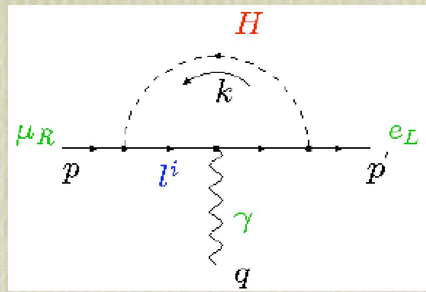
taken from Agashe, Perez, Soni, 2004

- anarchical flavor structure:
  - in diagonal bulk mass basis, 5D Yukawa matrices have no structures, i.e. all elements of the same order
- $\Lambda > 21$ TeV
- Generally: $\Lambda > O(10)$ TeV   Csaki, Falkowski & Weiler, 2008

Lepton Flavor Violations

Contributions from FCNCs:

- presence even in the limit of massless neutrinos
- at tree level:
  - tri-lepton decays (μ → 3 e, etc), μ-e conversion



Contributions from charged currents:

- at one-loop: $\mu \to e + \gamma$

Agashe, Blechman, Petriello, 2006

➡ Constraint on cutoff scale: $\Lambda_{\mu\text{-}e} > 5.9$ TeV

➡ **tension between tree-level and one-loop processes:**

- opposite dependence on Yukawa couplings
- tree-level FCNC $\sim \dfrac{1}{\lambda_{5D}}$
- one-loop charged current contributions $\sim \lambda_{5D}^2$

Minimal Flavor Violation

D'Ambrosio, Giudice, Isidori, Strumia, 2002
Cirigliano, Grinstein, Isidori, Wise, 2005

- assume Yukawa couplings the only source of flavor violation
- SM: absence of Yukawa couplings (with massless neutrinos}

$$G_F \equiv \mathrm{SU}(3)_q^3 \otimes \mathrm{SU}(3)_\ell^2 \qquad \begin{aligned} \mathrm{SU}(3)_q^3 &= \mathrm{SU}(3)_{Q_L} \otimes \mathrm{SU}(3)_{U_R} \otimes \mathrm{SU}(3)_{D_R} \\ \mathrm{SU}(3)_\ell^2 &= \mathrm{SU}(3)_{L_L} \otimes \mathrm{SU}(3)_{E_R} \ . \end{aligned}$$

- promote Yukawa couplings to be auxiliary fields

$$Y_U \sim (3, \bar{3}, 1)_{\mathrm{SU}(3)_q^3} \ , \qquad Y_D \sim (3, 1, \bar{3})_{\mathrm{SU}(3)_q^3} \ , \qquad Y_E \sim (3, \bar{3})_{\mathrm{SU}(3)_\ell^2}$$

- can rotate to

$$Y_D = \lambda_d \ , \qquad Y_L = \lambda_\ell \ , \qquad Y_U = V^\dagger \lambda_u$$

V: CKM matrix

- effects of flavor violation:

$$(\lambda_{\mathrm{FC}})_{ij} = \left(Y_U Y_U^\dagger\right)_{ij} \approx \lambda_t^2 V_{3i}^* V_{3j} \qquad i \neq j$$

- in RS model:
- the MFV assumption relates 5D Yukawa matrices & bulk mass terms
- implementation in quark sector

Fitzpatrick, Perez, Randall, 2007

MFV in Lepton Sector -- Massless Neutrinos

M.-C.C, H.B.Yu, 2008

Massless neutrino case:

- relevant 5D Lagrangian

$$\mathcal{L}_{5D}^{\text{lep}} \supset \overline{L} C_L L + \overline{e} C_e e + \overline{H}\, \overline{L} Y_e e$$

- Implementation of MFV:
  - only sources of flavor violation are Yukawa couplings

$$C_e = a Y_e^\dagger Y_e,\ C_L = b Y_e Y_e^\dagger$$

  - parameters a & b: O(1) proportionality constants

- MFV allows simultaneous diagonalization of C_e, C_L and Y_e

MFV in Lepton Sector -- Massless Neutrinos

- simultaneous diagonalization:
  - field transformations

$$L \to V\,L, \quad e \to W\,e \qquad \text{thus} \quad V^\dagger Y_e W \to \hat{Y}_e$$

$$V^\dagger Y_e Y_e^\dagger V \to \hat{Y}_e \hat{Y}_e^\dagger, \quad W^\dagger Y_e^\dagger Y_e W \to \hat{Y}_e^\dagger \hat{Y}_e$$

  - diagonal 5D Yukawa: $\hat{Y}_e = \mathrm{diag}(Y_{e_1}, Y_{e_2}, Y_{e_3})$
  - diagonal bulk masses: $C_e = a\hat{Y}_e^\dagger \hat{Y}_e$ and $C_L = b\hat{Y}_e \hat{Y}_e^\dagger$

- NO tree-level FCNCs
- intrinsically different from the anarchical assumption

$$\hat{Y}_e = \mathrm{diag}(Y_{e_1}, Y_{e_2}, Y_{e_3})$$

MFV in Lepton Sector -- Massless Neutrinos

- charged lepton masses

$$m_l \simeq v F_L Y_e F_e$$

F_L and F_e are the values of the zero-mode profiles on the TeV brane

- eigenvalues of F_L and F_e:

$$f_{L_i} = \sqrt{\frac{1-2c_{L_i}}{1-\epsilon^{1-2c_{L_i}}}},\ f_{e_i} = \sqrt{\frac{1-2c_{e_i}}{1-\epsilon^{1-2c_{e_i}}}}$$

$$\epsilon = e^{-\pi k r_c} \simeq 10^{-15}$$

c_{L_i} and c_{e_i} are eigenvalues of the 5D bulk mass C_L and C_e.

MFV in Lepton Sector -- Massless Neutrinos

- numerical results:

$$
\begin{aligned}
Y_e &= \mathrm{diag}(Y_{e_1}, Y_{e_2}, Y_{e_3}),\\
C_L &= \mathrm{diag}(b|Y_{e_1}|^2, b|Y_{e_2}|^2, b|Y_{e_3}|^2)\\
C_e &= \mathrm{diag}(a|Y_{e_1}|^2, a|Y_{e_2}|^2, a|Y_{e_3}|^2).
\end{aligned}
$$

$$a = 1 \text{ and } b = 1$$

$$Y_{e_1} \simeq 0.816, \; Y_{e_2} \simeq 0.759 \text{ and } Y_{e_3} \simeq 0.720,$$

- resulting charged lepton masses

$$m_e \simeq 0.511 \text{ MeV}, \; m_\mu \simeq 105.6 \text{ MeV and } m_\tau \simeq 1.77 \text{ GeV}$$

MFV in Lepton Sector -- Massive Neutrinos

M.-C.C, H.B.Yu, 2008

Massive neutrino case:

- introduce three RH neutrinos
- small Dirac neutrino masses by localizing RH neutrinos toward Planck brane

- relevant 5D Lagrangian

$$\mathcal{L}_{5\mathrm{D}}^{\mathrm{lep}} \supset \overline{L} C_L L + \overline{e} C_e e + \overline{N} C_N N + \overline{H}\, \overline{L} Y_e e + H \overline{L} Y_\nu N$$

- Implementation of MFV:
  - only sources of flavor violation are Yukawa couplings

$$C_e = a Y_e^\dagger Y_e, \quad C_N = d Y_\nu^\dagger Y_\nu, \quad C_L = c(\xi Y_\nu Y_\nu^\dagger + Y_e Y_e^\dagger)$$

  - parameters a & c & d: O(1) proportionality constants

MFV in Lepton Sector -- Massive Neutrinos

- can rotate to basis where either Y_e or Y_ν is diagonal
- without loss of generality: work in Y_e diagonal basis $Y_e = \hat{Y}_e$

$$Y_\nu = V_{5D} \hat{Y}_\nu, \qquad V_{5D}: \quad \text{5D Leptonic Mixing Matrix}$$

- in this basis, both C_N and C_e are diagonal, but not C_L

$$\hat{C}_N \equiv d\hat{Y}_\nu \hat{Y}_\nu^\dagger \text{ and } \hat{C}_e \equiv a\hat{Y}_e \hat{Y}_e^\dagger$$

$$C_L \simeq (\xi V_{5D} \hat{C}_N V_{5D}^\dagger + \hat{C}_e)$$

- eigenvalues of C_L: zero mode localizations of SU(2) doublets
- leads to a set of constraints on 5D bulk mass parameters

MFV in Lepton Sector -- Massive Neutrinos

- non-diagonal term ~ ξ in C_L: source of FCNC in charged lepton sector
- contributions to FCNC: depends on ξ
- V_{5D} unknown: taking the trace:

$$\mathrm{Tr}(C_L) \simeq c(\xi \mathrm{Tr}(C_N) + \mathrm{Tr}(C_e))$$

- small neutrino masses ⇒ small ξ
- realistic charged lepton masses:

$$C_{L_i},\, C_{e_i} \sim (0.4 - 0.6)$$

- small neutrino masses: RH neutrinos close to Planck brane

$$C_{N_i} \sim (1.2 - 1.5)$$

- The trace relation then implies $\xi \sim (0 - 0.1)$
- FCNC contributions suppressed by $\xi^2 \sim O(0 - 10^{-2})$

Charged Current Contributions

- in the presence of massive neutrinos:
  - charged current contributions to LFV

- MFV does not suppress charged current contributions to LFV

Numerical Results (massive neutrino case)

$$\xi \simeq 0 \qquad a = c = d = 4.$$

$$Y_{e_1} \simeq 0.405, \quad Y_{e_2} \simeq 0.375 \qquad Y_{e_3} \simeq 0.354,$$

$$\theta_{12} \simeq 1.383, \quad \theta_{23} \simeq 1.358, \quad \theta_{13} \simeq 1.338,$$

$$Y_{\nu 1} \simeq 0.713, \ Y_{\nu 2} \simeq 0.5634 \ \text{and} \ Y_{\nu 3} \simeq 0.5475,$$

$$\hat{Y}_\nu = \mathrm{diag}(Y_{\nu_1}, Y_{\nu_2}, Y_{\nu_3})$$

$$Y_\nu \equiv V_{5D}\hat{Y}_\nu \simeq \begin{pmatrix} 0.0307 & 0.128 & 0.533 \\ -0.275 & -0.504 & 0.123 \\ 0.657 & -0.217 & 0.0267 \end{pmatrix}.$$

- resulting masses and mixing angles:

$$\sin^2\theta^\nu_{12} \simeq 0.28, \quad \sin^2\theta^\nu_{23} \simeq 0.49, \quad \sin^2\theta^\nu_{13} \simeq 0.023$$
$$\Delta m^2_{21} \simeq 7.4 \times 10^{-5}\mathrm{eV}^2, \quad \Delta m^2_{31} \simeq 2.7 \times 10^{-3}\mathrm{eV}^2$$

- for $\mathrm{Br}(\mu \to e\gamma) \sim 10^{-12}$ 1st KK mass ~ 3 TeV allowed

Comments

- mild hierarchy among 5D parameters: ~ O(25) needed for large neutrino mixing

$$Y_\nu \equiv V_{5D}\hat{Y}_\nu \simeq \begin{pmatrix} 0.0307 & 0.128 & 0.533 \\ -0.275 & -0.504 & 0.123 \\ 0.657 & -0.217 & 0.0267 \end{pmatrix}.$$

generic anarchy case:

$$V_{ij} \sim f_{L_i}/f_{L_j}$$

large atm & solar mixing angles: $f_{L_1}/f_{L_2} \sim 1$ and $f_{L_2}/f_{L_3} \sim 1$.

- MFV with $\xi = 0$, f_{L_i}/f_{L_j} is fixed by $\sqrt{m_i/m_j}$

$$f_{L_1}/f_{L_2} \simeq 0.07 \qquad f_{L_2}/f_{L_3} \simeq 0.24.$$

- some structure in 5D Yukawa needed to accommodate mixing angles and mass ratios simultaneously

Comments

- counting the number of independent parameters that determine 5D Yukawa and bulk masses in lepton sector: (without CPV)
  - anarchical case: 27 parameters
    - 3x3 (diagonal C) + 2 x 9 (general Y)

$$\mathcal{L}_{\rm 5D}^{\rm lep} \supset \overline{L} C_L L + \overline{e} C_e e + \overline{N} C_N N + \overline{H}\, \overline{L} Y_e e + H \overline{L} Y_\nu N$$

  - MFV: 12 parameters
    - 3x3 (eigenvalues of Y) + 4 (prop. constants) - 1 (trace relation)

$$Y_\nu = V_{5D} \hat{Y}_\nu,$$

$$\hat{C}_N \equiv d \hat{Y}_\nu \hat{Y}_\nu^\dagger \text{ and } \hat{C}_e \equiv a \hat{Y}_e \hat{Y}_e^\dagger$$

- allow suppression in FCNC & charged current contributions to LFV
  - not possible in anarchical case, due to opposite dependence on 5D Yukawas

Conclusions

- RS model: provides novel way to generate fermion mass hierarchy
  - additional sources of flavor violation: the bulk parameters
  - leads to tree-level FCNC
  - tension between tree- and one-loop contributions
  - generically, $\Lambda > O(10)$ TeV
- with Minimal Flavor Violation:
  - tree-level FCNCs suppressed due to small neutrino masses
  - massless neutrino limit: NO tree-level FCNCs
- solutions exist that give realistic lepton masses and mixing angles
- 1st KK mass ~ 3 TeV allowed
  - viable solution to gauge hierarchy problem
  - testability at collider experiments